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**3 SEM TDC PHYH (CBCS) C 5**

**2021**

( Held in January/February, 2022 )

PHYSICS

( Core )

Paper : C–5

**( Mathematical Physics—II )**

*Full Marks* : 53
*Pass Marks* : 21

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Choose the correct answer : 1×5=5

*(a)* The value of $erf_c(x) + erf_c(-x)$ is

*(i)* 1

*(ii)* 2

*(iii)* –1

*(iv)* 0



( Turn Over )

(b) The value of $\Gamma\left(-\frac{1}{2}\right)$ is

(i) $\sqrt{\pi}$

(ii) $\frac{-\pi}{2}$

(iii) $-2\sqrt{\pi}$

(iv) 0

(c) The value of Legendre polynomial $P_2(x)$ is

(i) $(1-3x^2)$

(ii) $\frac{1}{2}(3x^2-1)$

(iii) $(3x^2-1)$

(iv) $\frac{1}{2}(1-3x^2)$

(d) The differential equation

$$\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2ny=0$$

is known as

(i) Legendre's equation

(ii) Bessel's equation

(iii) Laguerre's equation

(iv) Hermite's equation



( Continued )

(e) The sum $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots$ is

(i) $\frac{\pi^2}{12}$

(ii) $\frac{\pi^2}{6}$

(iii) $\frac{\pi^2}{8}$

(iv) None of the above

**2.** (a) State the Dirichlet's conditions for a Fourier series. 2

(b) Expand the function $f(x) = x \sin x$ in a Fourier series in the interval $-\pi \le x \le \pi$. Hence show that

$$\frac{1}{1\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}-\cdots=\frac{\pi-2}{4}$$ 4+2=6

(c) Expand $f(x) = e^x$ in a cosine series over (0, 1). 3

**3.** (a) What do you mean by ordinary and singular points of a differential equation? Find the nature of the point $x = -1$ with reference to the differential equation

$$x^2(x+1)\frac{d^2y}{dx^2}+(x^2-1)\frac{dy}{dx}+2y=0$$ 1+2=3

(b) Solve the following using Frobenius method (any *one*) : 5

(i) $9x(1-x)y'' - 12y' + 4y = 0$

(ii) $xy'' + y + xy = 0$

(c) Express $2 - 3x + 4x^2$ in terms of Legendre polynomials. 3

*Or*

Prove that

$$\int_{-1}^{1} xP_n(x)\,P_{n-1}(x)\,dx = \frac{2n}{4n^2 - 1}$$

(d) Evaluate the following : 2+2=4

(i) $P_n(1)$

(ii) $\int_{-1}^{1} P_3^2(x)\,dx$

**4.** Evaluate : 3

$$\int_0^{\infty} x^{n-1} e^{-h^2x^2}\,dx$$

*Or*

Show that

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

**5.** Answer any *two* of the following : 3×2=6

*(a)* Find the absolute error, relative error and percentile error when 754126 is rounded to four significant digits.

*(b)* If $u = \frac{5x^3y^4}{z^5}$ and errors in $x$, $y$, $z$ be 0·001, compute the relative maximum error when $x = y = z = 1$.

*(c)* State and prove the normal law of errors.

**6.** *(a)* Solve any *two* of the following partial differential equations by method of separation of variables : 4×2=8

*(i)* $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

*(ii)* $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$, under the conditions

$$u = 0 \text{ at } t = 0; \frac{\partial u}{\partial t} = 0 \text{ at } x = 0$$

*(iii)* $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$, under the conditions

$$\text{at } x = 0, u = 0 \text{ and } \frac{\partial u}{\partial x} = 1 + e^{-3y}$$

(b) Find the solution of one-dimensional wave equation in Cartesian coordinates. 5

*Or*

Find the solution of 2-D Laplace's equation in cylindrical coordinates.

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