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1 SEM TDC MTMH (CBCS) C 1

2021

(Held in January/February, 2022)

MATHEMATICS

(Core)

Paper : C–1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Write the value of $\frac{d}{dx}(\cosh x)$. 1

(b) Inverse hyperbolic sine is symmetric about a line. Write that line. 1

(c) Write the value of y_n, if $y = \cos(4x + 3)$. 1

(d) Define point of inflection. 1

(e) Find $\frac{d}{dx}(\tanh\sqrt{1 + x^2})$. 2

(f) Show that $\sinh x$ is an increasing function of x. 2

(g) Show that $y = x^2$ is concave up on $(-\infty, \infty)$. 2

(h) Show that $\text{cosech}^{-1} x = \sinh^{-1} \frac{1}{x}$. 3

Or

Find the asymptotes of

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0$$

(i) Find y_n, if $y = \sin^3 x$. 3

Or

Find y_n, if $y = x^3 \sin x$.

(j) Evaluate (any *one*) : 4

(i) $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

(ii) $\lim_{x \to \frac{\pi}{2}} \frac{\tan 5x}{\tan x}$

2. (a) Find $\int \tan^5 x \, dx$. 3

Or

Evaluate $\int_0^1 x^2 (1 - x)^{\frac{3}{2}} dx$.

(b) Obtain the reduction formula for

$$\int \sin^n x \, dx$$ 4

(c) Obtain the reduction formula for

$$\int x^n e^{ax} \, dx$$ 4

(Continued)

Or

Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

(d) Find the volume of the solid generated by revolving the region bounded by the curves and lines $y = x$, $y = -\frac{x}{2}$, $x = 2$ about the y-axis. 4

3. *(a)* Write the equation $x^2 + y^2 = 1$ in parametric form. 1

(b) A function $y = f(x)$ is defined on $[a, b]$. Write the domain of the function after given a natural parametrization

$$x = t,\ y = f(t)$$ 1

(c) Write the parametric formula for $\frac{d^2y}{dx^2}$. 1

(d) Write the equivalent Cartesian equation of the polar equation $r\cos\theta = 2$. 1

(e) Find the eccentricity of the ellipse $2x^2 + y^2 = 2$. 2

(f) Find the polar equation of $xy = 1$. 2

(g) Find the Cartesian equation from the parametric equation

$$x = 4\cot t,\ y = 2\sin t,\ 0 \le t \le 2\pi$$ 3

(h) Find a parametrization for the curve having the lower half of the parabola $x-1=y^2$. 4

Or

Find an equation for the line tangent to the curve $x=2\cos t$, $y=2\sin t$ at the point $t=\frac{\pi}{4}$.

4. (a) Define limit of a vector valued function. 1

(b) Let the position of a moving particle is given by

$$\vec{r}(t)=(\sec t)\hat{i}+(\tan t)\hat{j}+\frac{t^3}{3}\hat{k}$$

Find the acceleration at any time t. 2

(c) Evaluate the integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}[(\sin t)\hat{i}+(1+\cos t)\hat{j}+(\sec^2 t)\hat{k}]\,dt$$ 3

(d) Write the value of $[\vec{a}\ \vec{b}\ \vec{a}]$. 1

(e) Let $\vec{U}(t)$ and $\vec{V}(t)$ are differentiable vector function of t. Show that

$$\frac{d}{dt}(\vec{U}\cdot\vec{V})=\vec{U}'\cdot\vec{V}+\vec{U}\cdot\vec{V}'$$ 3

Or

Find the normal component of acceleration of a moving particle.

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