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**1 SEM TDC GEMT (CBCS)
GE 1 (A/B/C)**

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(March)

MATHEMATICS

(Generic Elective)

*The figures in the margin indicate full marks
for the questions*

Paper : GE-1 (A)

(**Differential Calculus**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) Define limit of a function. 1
(b) Choose the correct answer : 1

A function $f(x)$ is said to be continuous
at $x = a$ if

(i) $\lim_{x \rightarrow a} f(x)$ exists

(ii) $f(a)$ exists

(iii) $\lim_{x \rightarrow a} f(x) = f(a)$

(iv) $\lim_{x \rightarrow a} f(x) \neq f(a)$

(c) Let f be the function given by

$$f(x) = \frac{x^2 - a^2}{x - a}, \quad x \neq a$$

Using (ϵ, δ) definition, show that

$$\lim_{x \rightarrow a} f(x) = 2a \quad 2$$

(d) If f is even and differentiable function, prove that $f'(x)$ is odd. 2

2. (a) If $y = \cos 3x$, write y_n . 1

(b) If $y = x^{2n}$, where n is a positive integer, show that

$$y_n = 2^n \{1 \cdot 3 \cdot 5 \cdots (2n-1)\} x^n \quad 3$$

(c) If $y = \cos(m \sin^{-1} x)$, show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0 \quad 5$$

Or

If $f(x) = \tan x$ and n is a positive integer, prove with the help of Leibnitz's theorem that

$$f(0)^n - {}^n C_2 f(0)^{n-2} + {}^n C_4 f(0)^{n-4} - \dots = \sin \frac{n\pi}{2}$$

- (d) Investigate the type of discontinuity of the function f at $x=1$ defined by

$$\begin{aligned} f(x) &= 5x+9, & x > 1 \\ &= 14x-9, & x < 1 \\ &= 14, & x = 1 \end{aligned}$$

4

Or

If $f(x)$ is continuous on an interval I , a and b are any two numbers of I , then show that if y_0 is a number between $f(a)$ and $f(b)$, there exists a number c between a and b such that $f(c) = y_0$.

3. (a) If

$$u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$$

find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

2

- (b) If $y = f(x+ct) + \phi(x-ct)$, show that

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

4

Or

If $u = e^{xyz}$, prove that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$$

- (c) State and prove Euler's theorem on homogeneous function of two variables. 5

Or

If

$$u = 2 \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$$

show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \cot \frac{u}{2} = 0$$

4. (a) Write the equation of normal to the curve $y = f(x)$ at the point (x_1, y_1) . 1
- (b) Define curvature of a curve at a point. 2
- (c) Find the equation of the tangent to the curve $y(x-2)(x-3) - x + 7 = 0$ at the point where it meets x -axis. 3

Or

Find the curve whose curvature at any point on it is zero.

- (d) With the help of a graph describe the motion of a particle whose position $P(x, y)$ at time t is given by parametric equations $x = a \cos t, y = b \sin t; 0 \leq t \leq 2\pi$. 5

Or

Find the asymptotes of the graph of

$$f(x) = \frac{x^2 - 3}{2x - 4}$$

5. (a) Define double point of a curve. 1
- (b) Find the Cartesian equation of the curve and identify the graph for the following polar equation : 3
- $$r^2 = 4r \cos \theta$$
- (c) Trace any *one* of the following curves : 5
- (i) $y = x^3 - 12x - 16$
- (ii) $r = 1 - \cos \theta$

6. (a) Choose the correct answer : 1
- "If f is continuous in $[a, b]$, derivable in (a, b) and $f(a) = f(b)$, then there exists at least one point $c \in (a, b)$ such that $f'(c) = 0$." This result is known as
- (i) Lagrange's theorem
- (ii) Euler's theorem
- (iii) Rolle's theorem
- (iv) Taylor's theorem
- (b) Write the remainder after n term of Taylor's theorem in Cauchy's form. 1
- (c) Verify Rolle's theorem for the following function : 3

$$f(x) = \frac{x^3}{3} - 3x, x \in [-3, 3]$$

- (d) If $f'(x) = g'(x)$ at each point of an interval I , then there exists a constant c . Prove that $f(x) = g(x) + c$, for all x in I . 3
- (e) State and prove Lagrange's mean-value theorem. 5

Or

Using Maclaurin's series expand $\log(1+x)$ in an infinite series in powers of x .

7. (a) Define critical point of a function. 1
- (b) Find the absolute extrema values of $f(x) = 8x - x^4$ on $[-2, 1]$. 4
- (c) Evaluate (any two) : $3 \times 2 = 6$

(i) $\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$

(ii) $\lim_{x \rightarrow \infty} \frac{x^4}{e^x}$

(iii) $\lim_{x \rightarrow 1} \left(\frac{1}{x^2 - 1} - \frac{2}{x^4 - 1} \right)$

(iv) $\lim_{x \rightarrow 0} (\sin x)^{2 \tan x}$

- (d) State and prove Taylor's theorem with Lagrange's form of remainder.

6

Or

If f is continuous in $[a, b]$ and possesses first and second derivatives for $x = x_0$, where $a < x_0 < b$, prove that

$$f''(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

Paper : GE-1 (B)

(Object-oriented Programming in C++)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. Answer any ten from the following questions : 1×10=10
- (a) What is a base class and a derived class?
 - (b) How does a class enforce data hiding?
 - (c) What is encapsulation?
 - (d) Write the different access specifiers in a class.
 - (e) What is an inline function?
 - (f) What is a parameterized constructor?
 - (g) Write the use of copy constructor.
 - (h) Write the importance of destructor.
 - (i) What is containership?
 - (j) Write the use of array.
 - (k) Enlist some advantages of OOP.
 - (l) What is a function prototype?

2. Answer any *three* from the following questions : 2×3=6

- (a) How are objects implemented in C++?
- (b) Write the advantage of operator overloading.
- (c) What is a reference variable? What is its usage?
- (d) What is the difference between type casting and automatic type conversion?
- (e) Write the output of the following code fragment :

```
int ch = 20;
cout << ch << ++ ch << "\n";
```

3. Answer any *three* from the following questions : 4×3=12

- (a) What is a member function? How does a member function differ from an ordinary function?
- (b) Write four characteristics of a constructor function used in a class.
- (c) What is a temporary instance of a class? What is its use and how is it created?

- (d) Write down the various situations when a copy constructor is automatically invoked.
- (e) How does the invocation of constructors differ in derivation of class and nesting of class?

4. Answer any *two* from the following questions : 7×2=14

- (a) How does the visibility mode control the access of the members in the derived class? Explain with example.
- (b) Explain different types of inheritances with example.
- (c) Explain how the inheritance influence the working of constructor and destructor. Give example.

5. Answer any *three* from the following : $6 \times 3 = 18$

Write C++ program for the following :

- (a) To demonstrate the use of constructor and destructor
- (b) To reverse a given integer
- (c) To demonstrate the access control in public derivation of a class
- (d) To keep a count of created objects using static members

Paper : GE-1 (C)

(**Finite Element Methods**)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) What is the concept of analysis in finite elements method? 1
- (b) Illustrate different types of finite elements and their uses in associated fields. 4

Or

Discuss briefly different types of partial differential equations used in the application of finite elements method.

- (c) Mention a particular initial boundary value problem and discuss it briefly in the field of finite element method. 4
- (d) Define a triangular finite element. Derive its formula in two-dimensional Cartesian region. 5
2. (a) Describe briefly about weighted residual methods. 3

Or

Find an expression for Galerkin collocation method.

- (b) What do you mean by element assemblage in finite elements method? Give an example. 3

Or

Write down the importance of the application of finite elements method over certain practical domains.

3. (a) Find an expression for variational methods. 3

Or

Using Greens' theorem, find the Euler's equation.

- (b) Write down the importance of shape functions in finite element methods. 4

Or

Discuss briefly about numerical methods used in solving partial differential equations.

- (c) State and prove Dirichlet problem for Laplacian operator. 5

Or

Find the equation for weighted residual methods.

4. (a) What do you mean by conforming elements in finite element methods? 2
- (b) Discuss about natural coordinates in the process of formulation of a linear Lagrange polynomial. 3
- (c) Define a triangular element. Find an expression for linear Lagrange polynomial in case of triangular element. 4
- (d) Find an interpolating function over an one-dimensional domain. 3

Or

Find a formulation for the solution of the boundary value problem ∇^2

$$u = 1, \quad |x| \leq 1, |y| \leq 1$$

$$u = 0,$$

on the boundary in case of triangular elements.

5. (a) Define weak derivatives with an example. 2
- (b) Find an expression for rectangular elements and hence deduce its stiffness equation. 3

Or

Discuss about degrees of freedom of a finite element. Draw a picture of mixed plane elements with three degrees of freedom.

- (c) Discuss about the importance of isoparametric elements in solving boundary value problems. 3
- (d) Write down the procedure of calculating sparse matrix with the help of an example. 4

Or

Solve the boundary value problem

$$u'' + (1 + x^2)u + 1 = 0, u(\pm 1) = 0$$

with linear piecewise polynomial for two elements of equal length.

6. (a) Define Hilbert space. 1
- (b) Find an expression for numerical integration over finite elements in one and two dimensions. 5
- (c) Write a note on convergence analysis and completeness in Galerkin finite element methods. 6

Or

Find an algorithm for developing a formulation in line element mesh generation.

7. (a) Answer any one of the following : 4

(i) Find the assembly element equations of a line element while solving linear boundary value problem.

(ii) A triangular element (e) is expressed as linear function of x and y as follows :

$$u^{(e)}(x, y) = a + bx + cy$$

Find its shape function.

(b) Derive the local stiffness matrix for plane elasticity for a four-node rectangular element. 4

(c) Derive the boundary value problem that characterizes the minima of the functional

$$J[u] = \frac{1}{2} \int_0^1 [(u'')^2 - 2(u')^2 + u^2 - 2u] dx$$

$$u(0) = u'(0) = 0, u(1) = u'(1) = 0 \quad 4$$

Or

Find the variational functional for the boundary value problem :

$$u'' = u - 4xe^x$$

$$u'(0) - u(0) = 1, u'(1) + u(1) = -e$$

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