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1 SEM TDC MTMH (CBCS) C 2

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(November)

MATHEMATICS

(Core)

Paper : C-2

(Algebra)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Find $|4+3i|$. 1

(b) Express $\frac{3-2i}{-1+i}$ in the form $x+iy$. 2

(c) If

$$\sin\alpha+\sin\beta+\sin\gamma=\cos\alpha+\cos\beta+\cos\gamma=0$$

prove that

$$\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$$

and

$$\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$$ 3

(d) Find the cube root of $8i$. 4

2. *(a)* State the well-ordering property of positive integers. 1

(b) Let $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\sin x$ and $g:\mathbb{R}\to\mathbb{R}$ given by $g(x)=x^2$. Then find $(f\circ g)(x)$ and $(g\circ f)(x)$. 1

(c) If $f:A\to B$ be one-one onto, then prove that inverse mapping of f is unique. 2

(d) Consider $N\times N$ be the set of orderrd pairs of natural numbers. Let $\mathcal{R}$ be the relation in $N\times N$ defined by $(a, b)\mathcal{R}(c, d)$ iff $a+d=b+c$. Then show that $\mathcal{R}$ is an equivalence relation. 3

(e) Show that $f:X\to Y$ be invertible iff f is a bijection. Also show that $g:\mathbb{R}\to\mathbb{R}$ defined by $g(x)=3x+5$ is a bijection and find its inverse. 3+2+1=6

(f) State and prove the division algorithm. 1+4=5

Or

Prove by mathematical induction that n^5-n is divisible by 30. 5

(g) Find the g.c.d. of 1166 and 256 and express it in the form

$$1166x+256y=(1166, 256)$$
4

(h) If $(a, c)=(b, c)=1$, then prove that $(ab, c)=1$. 3

3. *(a)* Express $v=(1, 3, 2)$ as a linear combination of $u_1=(1, 2, 1)$, $u_2=(2, 6, 5)$ and $u_3=(1, 7, 8)$. 2

(b) Solve : 2

$$2x-6y+7z=1$$
$$4y+3z=8$$
$$2z=4$$

(c) What do you mean by linear dependence of vectors? Show that the vectors $(1, 2, -3, 4)$, $(3, -1, 2, 1)$ and $(1, -5, 8, -7)$ are linearly dependent. 1+3=4

(d) Find for what values of λ and μ, the system of equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

has *(i)* no solution, *(ii)* a unique solution and *(iii)* an infinite number of solutions. 2+2+2=6

(e) Find the row-reduced echelon form of

$$A=\begin{bmatrix}2 & -1 & 3\\ 3 & 2 & 1\\ 1 & -4 & 5\end{bmatrix}$$

and hence find the rank of it. 6

Or

Reduce $A = \begin{bmatrix} \frac{1}{3} & \frac{3}{2} & 3 & -8 \\ \frac{2}{3} & 3 & 2 & 1 \\ 1 & \frac{9}{2} & 5 & -7 \\ \frac{4}{3} & 6 & 8 & -15 \end{bmatrix}$ to its normal form.

4. (a) Let $F: \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$F(x, y) = (x - y, x + 2y)$$

Determine whether F is singular or non-singular. 2

(b) Find the matrix representation of the linear transformation T on $\mathbb{R}^3$ given by $T(x, y, z) = (x, y, 0)$. 3

(c) Find the eigenvalues of

$$A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

3

(d) Let

$$P = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

(i) Find the eigenvectors corresponding to the eigenvalues.

(ii) Find the characteristic polynomial of P. 2+2=4

(e) Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$$

5

Or

Find a linear map $T: \mathbb{R}^3 \to \mathbb{R}^4$ whose image is spanned by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$.

(f) Show that a matrix A and its transpose A^T have the same characteristic polynomial. 3

Or

Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$. Find $f(A)$, where $f(t) = t^2 - 3t + 7$.

(g) Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors $u_1 = (1, -2, 5, -3)$, $u_2 = (2, 3, 1, -4)$, $u_3 = (3, 8, -3, -5)$. Find a basis and dimension of W. 5

Or

Determine whether $(1, 1, 1, 1)$, $(1, 2, 3, 2)$, $(2, 5, 6, 4)$, $(2, 6, 8, 5)$ form a basis of $\mathbb{R}^4$. If not, find the dimension of subspace they span.

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