Total No. of Printed Pages—7

5 SEM TDC PHY M 1

2021

(March)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24/18

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following (any *five*) : 1×5=5

(a) A Fourier series of a function $f(x)$ contains only cosine terms, if function $f(x)$ is

(i) an odd function of x

(ii) an even function of x

(iii) an exponential function containing real terms only

(iv) It is not possible

(b) What is the sum of residues of the function $f(z) = \frac{e^z}{z^2 + a^2}$ at all its poles?

(i) $\frac{\sin a}{a}$

(ii) $-\frac{\sin a}{a}$

(iii) $\frac{\cos a}{a}$

(iv) $-\frac{\cos a}{a}$

(c) The general solution of the ordinary differential equation $\frac{d^2y}{dx^2} + 4y = 0$ is

(i) $Ae^{2x} + Be^{-2x}$

(ii) $(A + Bx)\, e^{-2x}$

(iii) $A\cos 2x + B\sin 2x$

(iv) $(A + Bx)\cos 2x$

(d) If z_1 and z_2 be two complex numbers, then $|z_1 \pm z_2|$ is

(i) $\leq |z_1| + |z_2|$

(ii) $< |z_1| - |z_2|$

(iii) $> |z_1| + |z_2|$

(iv) $\geq |z_1| + |z_2|$

(e) Using Fourier integral formula, find the value of

$$\frac{2a}{\pi}\int_0^{\infty}\frac{\cos\lambda x}{\lambda^2+a^2}d\lambda \ (a>0)$$

(i) 1

(ii) e^{ax}

(iii) e^{-ax}

(iv) None of the above

(f) $\Gamma\left(-\frac{3}{2}\right)$ is equal to

(i) $\frac{3\pi}{4}$

(ii) $-\frac{3\pi}{4}$

(iii) $\frac{3\pi}{4}\sqrt{\pi}$

(iv) $\frac{3\pi}{2}\sqrt{\pi}$

(g) Power series solution is applicable to differential equations which are

(i) second order of degree n

(ii) partial differential equations

(iii) linear homogeneous

(iv) None of the above

2. Answer any *five* of the following : 2×5=10

(a) Show that

$$P_n(-x) = (-1)^n P_n(x)$$

(b) State Fourier's theorem and Dirichlet condition.

(c) Examine whether $\sin z$ is an analytic function of z.

(d) Prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

(e) Expand in Fourier series the function $f(x) = x$ in the interval $-1 < x < 1$.

(f) Solve the following differential equation :

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0;\ y(0) = 2,\ \frac{dy}{dx} = 1$$

(g) Using Cauchy's integral formula, calculate the integral

$$\int_C \frac{zdz}{(9 - z^2)(z + i)}$$

where C is the circle $|z| = 2$ described in the positive sense.

3. *(a)* Find the solutions of the equation

$$\frac{d^2y}{dx^2} + w^2 y = 0$$

using Frobenius method. 5

Or

Solve in series the equation

$$\frac{d^2y}{dx^2} + x^2 y = 0$$

(b) Show that

$$\int_{-1}^{+1} [P_n(x)]^2 \, dx = \frac{2}{2n+1}$$ 5

(c) Show that

$$\Gamma m \, \Gamma(1-m) = \frac{\pi}{\sin m\pi}$$ 4

(d) Solve

$$(3x + 2y^2) y \, dx + 2x(2x + 3y^2) \, dy = 0$$ 4

(e) Show that for $|x|$ is large

$$\text{erfc}(x) = 1 - \text{erf}(x)$$ 3

4. *(a)* State and prove Cauchy's residue theorem. 4

(b) Find the Taylor series expansion of a function of the complex variable

$$f(z) = \frac{1}{(z-1)(z-3)} \text{ about the point } z = 4.$$

Also, find its region of convergence. 4

Or

Apply the method of contour integration to evaluate

$$\int_0^\infty \frac{1-\cos x}{x^2} dx$$

(c) What is an analytic function? Derive the necessary condition for a function to be analytic. 1+3=4

5. *(a)* A sawtooth wave is given by

$$f(x) = x \text{ for } -\pi \le x \le \pi$$

Show that

$$f(x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n}$$

Also, plot the graphical representation of the function $f(x)$ in the interval $[-\pi, \pi]$ and its periodic extension outside $[-\pi, \pi]$. 3+1=4 *(b)* Obtain Fourier series for the expansion $f(x) = x\sin x$ in the interval $-\pi < x < \pi$. Hence deduce that

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots$$ 4

(c) Expand the function $f(x) = \sin x$ as a cosine series in the interval $(0, \pi)$. 4

Or

Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.

★★★